# Tax incidence on competing two-sided platforms*

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**Abstract**

We analyze the effects of various taxes on competing two-sided platforms. First we consider non-discriminating taxes. We show that specific taxes are entirely passed to the agents on the side on which they are levied; other agents and platforms are left unaffected. Transaction taxes hurt agents on both sides and benefit platforms. Ad valorem taxes are the only tax instrument that allows the tax authority to capture part of the platforms' profits. Second, regarding asymmetric taxes, we show that agents on the untaxed side benefit from the tax. At least one platform, possibly the taxed one, benefits from the tax.

*Keywords:* Two-sided platforms, specific tax, ad valorem tax, transaction tax

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# 1 Introduction

Two-sided platforms intermediate between two distinct groups of economic agents that benefit from interacting with one another but fail to organize this interaction by their own forces because of high transaction costs. That is, the main function of two-sided platforms is to internalize the various external effects that the interaction between the two groups generate. Of particular interest are the cross-side effects that make the well-being of one group depend on the participation of the other group: in general, the more one group participates, the more valuable the platform becomes for the other group.

Two-sided platforms are active in a large variety of settings, as exemplified by the following categories: *hardware & software systems* allow applications developers and end users to interact (e.g., Mac OS, Android, PlayStation); *transaction systems* provide a method for payment to buyers and sellers that are willing to use it (e.g., Visa, Bitcoin, PayPal); *matchmakers* help members of one group to find the right 'match' within another group (e.g., Alibaba, Monster, Meetic); *exchanges* help 'buyers' and 'sellers' search for feasible contracts and for the best prices (e.g., eBay, Booking.com, Wiley, edX); *crowdfunding platforms* allow entrepreneurs to raise funds from a 'crowd' of investors (e.g., Kickstarter, Indiegogo, LendingClub); *peer-to-peer marketplaces*, facilitate the exchange of goods and services between 'peers' (e.g., Airbnb, Uber, EatWith, TaskRabbit); *digital media* (including *social networks* and *search engines*) provide content to users and sell users' attention to advertisers (e.g., YouTube, Facebook, Google Search).

As the previous examples illustrate, a large number of platforms have developed (or created) their business through the Internet and the use of digital technologies. Some of these 'digital platforms' have exploited the self-reinforcing nature of network effects, together with the global reach of the Internet, to become dominant players in many countries. One thinks, in particular, of the well-established 'GAFAM' and the emerging 'NATU'.[1] These companies are well-known to generate very large profits but to pay, comparatively, very low effective corporate taxes. They are accused, especially in Europe, to 'dodge' taxes by locating their activities and their profits in lower-taxed countries.

Given the difficulty to tax the corporate income of global digital platforms, several countries are thinking of using other tax instruments. For instance, in 2015, the UK government announced its intention to require business intermediaries and electronic payment providers to hand over information about their users, so as to identify individuals running a business through a digital platform and failing to declare a source of income. At the same time, the UK government also made public its project to introduce a tax relief that would make the tax position more certain and simple for the same individuals. In October 2016, French lawmakers voted in favor of a so-called "YouTube tax" that would apply a 2% levy on all streaming video.[2]

[1] GAFA stands for Google, Amazon, Facebook, Apple and Microsoft,while NATU stands for Netflix, Airbnb, Tesla and Uber.

[2] See `http://bit.ly/1hLYyOV`, `http://bit.ly/2hVjYKr` and `http://for.tn/2elgiAR` (last consulted 20/12/2016).

In contrast with a corporate tax, these alternative taxes directly affect the competitive game among platforms. Indeed, if the access to the platform and/or the transactions conducted on the platform are taxed, then platforms will react by modifying the prices they set for their services. How? Answering this question is far from obvious given the two-sided nature of these platforms. Indeed, when choosing its prices, platforms need to reflect not only the external effects among the groups of users that they serve, but also the strategic interactions with the rival platforms.

The objective of this paper is precisely to deepen our understanding of tax incidence on competing, and potentially asymmetric, two-sided platforms. To this end, we develop a model of Hotelling competition between two two-sided platforms. The two platforms serve the same two groups of agents; in each group, an agent's utility increases with the number of agents of the other group she can interact with (positive cross-side external effects). Platforms simultaneously set an access price for each group and upon observing these prices, agents choose which platform to visit (they are restricted to 'singlehome', i.e., to visit a single platform). Our strategy is then to analyze how various taxes affect the subgame-perfect equilibrium of this game.

Given the complexity of the model, we perform two different exercises. First, we want to compare the tax incidence of different symmetric taxes, i.e., taxes that are imposed on the two platforms in a non discriminatory way. We study in turn the effects of specific taxes, ad valorem taxes, and transaction taxes. The first two taxes concern the access to the platform, while the latter concern the usage of the platform. For instance, in the case of hardware/sofware systems, specific (unit) or ad valorem (percentage) taxes are levied on digital devices (such as smartphones or game consoles), whereas transaction taxes are levied on applications or on digital content (like the French "YouTube tax").[3] To keep the model tractable, we need to assume that platforms are symmetric, so that taxes only affect equilibrium prices but leave equilibrium participations unchanged. In this setting, we show the following. Specific taxes are entirely passed to the agents on the side on which they are levied; the agents on the other side and the platforms are left unaffected. Transaction taxes hurt agents on both sides and benefit platforms. As for ad valorem taxes, the only clear result is that a tax levied on one side hurts the agents on the other side; the taxed agents may benefit from the tax. If the tax authority seeks to find a way to compensate for ineffective corporate taxes, it should prefer ad valorem taxes. Our model shows indeed that it is the only tax instrument that captures part of the platforms' profits.

In our second exercise, we examine the effects of asymmetric taxes. We have in mind situations where the tax authority would be more efficient in collecting taxes from one platform than from the other. This may be due to the fact that platforms are located in different jurisdictions or that one platform collaborates more actively with the tax authority. In comparison with the first exercise, an asymmetric tax implies two major differences: first, the tax affects not only prices but also participations at equilibrium; second, the tax generates strategic effects as it allows one platform to commit to change its price structure. These two differences introduce

[3] In the short-term hosted accommodation industry, an occupancy tax that has to be paid per accommodated guest is a transaction tax; examples of specific and ad valorem taxes would be, respectively, a fixed amount per listed accomodation, and a percentage of the fee that hosts pay to register on the platform.

additional complexities, which force us to focus on specific taxes only so as to keep the model tractable. In particular, we assume that one of the two platforms has to pay a specific tax per agent on one side. The main results we derive from this setting are the following. All agents on the untaxed side benefit from the tax. The sum of platforms' profits is increased so that at least one platform benefits from the tax. Interestingly, the taxed platform could welcome the tax because of the strategic commitment it confers. We also show that agents on the taxed side may suffer from the tax but they may also benefit. In the latter case, the introduction of the tax improves welfare.

To the best of our knowledge, our analysis and our results are novel. This is partly explained by the fact that the literature analyzing the competition between two-sided platforms–following the seminal contributions of Caillaud and Jullien (2003), Rochet and Tirole (2003 and 2006), and Armstrong (2006)–has mostly considered symmetric platforms. Yet, a symmetric setting proves inadequate to examine the impact of taxes on competing platforms.

There are, however, a few papers that consider the issue of taxation in two-sided markets. Still, they do so in different settings or with different focus than ours. Kind, Koethenbuerger and Schjelderup (2008, 2009) are mainly concerned by comparing the impacts of ad valorem and unit taxes on tax revenues and on welfare in a monopoly two-sided market (with a specific focus on advertising-financed media). Some of the results of Kind, Koethenbuerger and Schjelderup (2010) echo ours; for instance, they show that a higher ad valorem tax on the user side does not necessarily induce the platform to raise the price on that side. Kind, Koethenbuerger and Stähler (2013) analyze the effects of taxes on newspaper differentiation. Kotsogiannis and Serfes (2010) address the issue of taxation of two-sided platforms in terms of tax competition between countries. Bloch and Demange (2017) focus on the effect of taxes on privacy protection (they model a monopolistic platform that collects data on users and make revenues either by exploiting this data or by selling it to third parties). Tremblay (2016) studies optimal taxation of a monopoly two-sided platform with two tax instruments (taxation on platform content and taxation on the platform itself). Bourreau, Caillaud and De Nijs (2017) assess the impacts of a tax on data collection and a tax on advertising on the pricing strategies of a monopoly two-sided platform, which offers personalized services to users and targeted advertising to sellers.

Our analysis also bears a clear connection with the (scarce) literature studying cost pass-through for multisided platforms or multiproduct firms (the unit tax we consider is indeed equivalent to a cost increase). Weyl (2010) analyzes cost pass-through for a monopoly two-sided platform, which is directly relevant to our analysis. However, our results cannot be compared as Weyl focuses on insulating tariffs (i.e., the platform is supposed to choose participation rates on the two sides rather than prices); the latter point makes a big difference as the effect of a cost (or tax) change on the price on side $i$ is computed under the assumption that participation is kept fixed on side $j$, which is not the case in our analysis.

As the interaction between the two sides generates strong complementarities, two-sided platforms bear some resemblance with multiproduct firms.[4] The studies of cost pass-through for

[4] Although, as Rochet and Tirole (2003) point out, end users internalize the corresponding externalities in a

multiproduct firms are thus insightful for our analysis. Moorthy (2005) analyzes a theoretical model where two competing retailers supply each two substitutable products to consumers, and examines how a cost increase for one firm affects this firm's prices, as well as its rival's prices. Alexandrov and Bedre-Defolie (2011), in contrast, suppose that the two retailers offer complementary products and that these products affect each other's demand in an asymmetric way (the price of one product influences the demand for the other product, but the reverse is not true); as will become clear below, our setting shares these two features. Armstrong and Vickers (2016) propose a general demand system for multiple products that yields simple formulas for the size and sign of own-cost and cross-cost passthrough relationships.

Finally, our result that a tax increase may raise profits of competing firms is not unheard of. For instance, this result is shown, e.g., by Hindriks and Myles (2006, Chapter 8) under Cournot competition and by Anderson, de Palma and Kreider (2001) under Bertrand competition and differentiated products. In the latter case (which is more relevant for this paper), the authors show that the profit increase can only happen for highly convex demands. As demands are linear in our model, the potential profit-enhancing effect of larger taxes clearly stems from a different channel.

The rest of the paper is organized as follows. Before examining tax incidence on prices on profits (Sections 3 and 4), we derive the equilibrium of a pricing game between two asymmetric platforms (Section 2). We discuss our results in Section 5.

# 2 Price competition between taxed platforms

We model the competition between two two-sided platforms in environments where agents of both sides can join at most one platform (so-called 'two-sided singlehoming').[5] We adopt the model of Armstrong (2006), which we extend to introduce various forms of taxes. Two platforms are located at the extreme points of the unit interval: platform $U$ (for *Uppercase*, identified hereafter by upper-case letters) is located at 0, while platform $l$ (for *lowercase*, identified by lower-case letters) is located at 1. Platforms facilitate the interaction between two groups of agents, noted $a$ and $b$. Both groups are assumed to be of mass 1 and uniformly distributed on $[0, 1]$. We analyze the subgame-perfect equilibria of the following two-stage game: first, platforms simultaneously set their access fees; second, agents decide which platform to visit.

multiproduct setting but not in a multisided setting.

[5] In the real world, singlehoming environments may result from indivisibilities or limited resources; for a discussion, see Case 22.4 in Belleflamme and Peitz (2015, p. 667).

**Agents' decisions.** We define the net utility functions for an agent of group $a$ and for an agent of group $b$, respectively located at $x$ and $y \in [0,1]$ as:

$$\begin{array}{ll} U_a(x) = R_a + \sigma_a N_b - \theta_a x - P_a & \text{if joining platform } U, \\ u_a(x) = R_a + \sigma_a n_b - \theta_a (1-x) - p_a & \text{if joining platform } l, \\ U_b(y) = R_b + \sigma_b N_a - \theta_b y - P_b & \text{if joining platform } U, \\ u_b(y) = R_b + \sigma_b n_a - \theta_b (1-y) - p_b & \text{if joining platform } l, \end{array}$$

where $R_j$ is the stand-alone benefit that agents of group $j$ derive from visiting any platform, $\sigma_j$ is the valuation for agents of group $j$ of the interaction with an additional agent of the other group (i.e., it measures the strength of the cross-side external effect exerted on agents of group $j$), $N_j$ (resp. $n_j$) is the mass of agents of group $j$ that decide to join platform $U$ (resp. $l$), $\theta_j$ is the 'transport cost' parameter for group $j$, and $P_j$ (resp. $p_j$) is the access fee that platform $U$ (resp. $l$) sets for agents of group $j$ (with $j$, $k \in \{a,b\}$ and $j \neq k$).

Let $\hat{x}$ (resp. $\hat{y}$) identify the agent of group $a$ (resp. $b$) who is indifferent between joining platform $U$ or platform $l$; that is, $U_a(\hat{x}) = u_a(\hat{x})$ and $U_b(\hat{y}) = u_b(\hat{y})$. Solving these equalities for $\hat{x}$ and $\hat{y}$ respectively, we have

$$\begin{array}{c} \hat{x} = \frac{1}{2} + \frac{1}{\theta_a}\left[\sigma_a\left(N_b - \frac{1}{2}\right) - \frac{1}{2}(P_a - p_a)\right], \\ \hat{y} = \frac{1}{2} + \frac{1}{\theta_b}\left[\sigma_b\left(N_a - \frac{1}{2}\right) - \frac{1}{2}(P_b - p_b)\right]. \end{array}$$

In what follows, we assume that each platform provides the agents with stand-alone benefits ($R_a$ and $R_b$) that are sufficiently large to make sure that all agents join one platform. Both sides are then 'fully covered', so that $N_j + n_j = 1$ ($j = a, b$). We maintain this assumption throughout our analysis. This means, in particular, that the tax levels that we will consider do not discourage any agent from visiting the platforms (i.e., taxes are assumed to leave total participation unaffected).

We can now use the fact that $\hat{x} = N_a = 1 - n_a$ and $\hat{y} = N_b = 1 - n_b$ to solve the above systems of equations for $N_a$ and $N_b$:

$$N_a = \frac{1}{2} + \frac{\theta_b}{2}\frac{p_a - P_a}{\theta_a\theta_b - \sigma_a\sigma_b} + \frac{\sigma_a}{2}\frac{p_b - P_b}{\theta_a\theta_b - \sigma_a\sigma_b}, \tag{1}$$

$$N_b = \frac{1}{2} + \frac{\theta_a}{2}\frac{p_b - P_b}{\theta_a\theta_b - \sigma_a\sigma_b} + \frac{\sigma_b}{2}\frac{p_a - P_a}{\theta_a\theta_b - \sigma_a\sigma_b}. \tag{2}$$

To ensure that participation on each side is a decreasing function of the access fee on this side, we assume that $\theta_a\theta_b > \sigma_a\sigma_b$. This assumption, which is common in the analysis of competition between two-sided platforms, says that the strength of cross-side external effects (measured by $\sigma_a\sigma_b$) is smaller than the strength of horizontal differentiation (measured by $\theta_a\theta_b$).

**Platforms' maximization problem.** Platforms simultaneously choose their access fees to maximize their profit. Let $\tau_k^v$ denote the rate of ad valorem (percentage) taxation on side $k$ and $\tau_k^s$, the level of the specific (unit) tax on side $k$. Let also $C_k$ and $c_k$ denote the marginal cost of serving an agent of group $k$ for, respectively, platform $U$ and $l$. We can then write platform $U$'s profit as[6]

$$\begin{aligned}\Pi &= \left(1-\tau_a^v\right)\left(P_a-\tau_a^s\right)N_a - C_aN_a + \left(1-\tau_b^v\right)\left(P_b-\tau_b^s\right)N_b - C_bN_b \\ &= \left(1-\tau_a^v\right)\left(P_a-\tilde{C}_a\right)N_a + \left(1-\tau_b^v\right)\left(P_b-\tilde{C}_b\right)N_b,\end{aligned} \tag{3}$$

where $\tilde{C}_k \equiv C_k/\left(1-\tau_k^v\right)+\tau_k^s$ is platform $U$'s *effective* cost on side $k$. We express platform $l$'s profit accordingly as

$$\pi = \left(1-\tau_a^v\right)\left(p_a-\tilde{c}_a\right)n_a + \left(1-\tau_b^v\right)\left(p_b-\tilde{c}_b\right)n_b, \tag{4}$$

with $\tilde{c}_k \equiv c_k/\left(1-\tau_k^v\right)+\tau_k^s$.

To derive the equilibrium prices, we first expand the above profit functions by taking advantage of the fact that $n_k = 1-N_k$, and by replacing $N_a$ and $N_b$ by expressions (1) and (2). We then need to solve the system made of the four first-order conditions for profit maximization: $\partial\Pi/\partial P_k = 0$ and $\partial\pi/\partial p_k = 0$ $(k=a,b)$. Unfortunately, the complexity of this problem does not allow us to come up with a general solution. In what follows, we therefore impose some simplifying assumptions and consider two separate issues. First, in Section 3, we assume that platforms are symmetric, which allows us to analyze fully the interplay between the various taxes. Second, in Section 4, we focus on specific taxes (and abstract away ad valorem taxes); thereby, we are able to examine the incidence of asymmetric taxes. In both cases, the second-order conditions are satisfied if[7]

$$\theta_a\theta_b > \tfrac{1}{4}\left(\sigma_a+\sigma_b\right)^2. \tag{5}$$

The latter condition can be interpreted as follows: the horizontal differentiation among platforms (measured by $\theta_a$ and $\theta_b$) must be large enough in comparison to the strength of the cross-side network effects (measured by $\sigma_a$ and $\sigma_b$); otherwise, platforms cannot coexist at equilibrium (agents on both sides would prefer to interact on a single platform).

# 3 Symmetric taxes

In this section, we restrict the attention to symmetric platforms; that is, we assume that the two platforms face exactly the same marginal costs: $c_k = C_k$ $(k=a,b)$. Also, as non-discrimination between firms in the same industry is a ground rule for taxation, we assume symmetric taxes.

[6] We follow Myles (1996) and Anderson *et al.* (2001) for the definition of taxes. This definition of the ad valorem tax rate is easier to work with than the one used, e.g., by Kind *et al.* (2008, 2009, 2010). In the usual formulation, ad valorem tax rates are expressed as a percentage of the producer price (i.e., the agent price on side $k$ is $(1+t_k)$ times the producer price, where $t_k$ is the ad valorem tax rate). The two formulations are made equivalent by setting $\tau_k^v = t_k/\left(1+t_k\right)$.

[7] The second order conditions require $\theta_a\theta_b > \sigma_a\sigma_b$ and $\theta_a\theta_b > \frac{1}{4}\left(\sigma_a+\sigma_b\right)^2$. We note that $\frac{1}{4}\left(\sigma_a+\sigma_b\right)^2 - \sigma_a\sigma_b = \frac{1}{4}\left(\sigma_a-\sigma_b\right)^2 > 0$, which means that the second condition is more stringent than the first.

A few lines of computations allow us to derive the equilibrium access fees as

$$\begin{aligned}
P_a^* &= p_a^* = \left[\tilde{C}_a + \theta_a\right] - \frac{1-\tau_b^v}{1-\tau_a^v}\sigma_b = \frac{C_a}{1-\tau_a^v} + \tau_a^s + \theta_a - \frac{1-\tau_b^v}{1-\tau_a^v}\sigma_b, \\
P_b^* &= p_b^* = \left[\tilde{C}_b + \theta_b\right] - \frac{1-\tau_a^v}{1-\tau_b^v}\sigma_a = \frac{C_b}{1-\tau_b^v} + \tau_b^s + \theta_b - \frac{1-\tau_a^v}{1-\tau_b^v}\sigma_a.
\end{aligned}$$

Equilibrium prices can be divided into two components. First there is the classic Hotelling formula, which is the addition of the (effective) marginal cost and the transport cost. Second, prices are adjusted downward to account for the positive indirect network effects. As Armstrong (2006) explains, the more agents on side $b$ value the interactions with the other group (i.e., for larger $\sigma_b$), the larger the incentive for the platform to lower the access fee for group $a$. Indeed, attracting more agents of group $a$ allows the platform to raise its revenues on side $b$. This strategy is all the more profitable that the ad valorem tax rate is lower on side $b$ than on side $a$.

Given that platforms are symmetric and set the same prices, agents on both sides split equally, which allows us to express the platforms' equilibrium profits as

$$\Pi^* = \pi^* = \tfrac{1}{2}\left[\left(1-\tau_a^v\right)\left(\theta_a - \sigma_a\right) + \left(1-\tau_b^v\right)\left(\theta_b - \sigma_b\right)\right] \tag{6}$$

The agents' surplus on side $k$ is

$$V_k^* \equiv \int_0^{N_k^*} U_k(x)\,dx + \int_{N_k^*}^1 u_k(x)\,dx$$

At the symmetric equilibrium, $N_k^* = 1/2$, $P_k^* = p_k^*$, and total transport costs are equal to $\theta_k/4$. Hence,

$$V_k = R_k + \tfrac{1}{2}\sigma_k - P_k^* - \tfrac{1}{4}\theta_k \tag{7}$$

We want now to examine who pays the various taxes. To this end, we compare the equilibrium with taxes (superscript $T$) to the equilibrium without any tax (superscript 0). We use expressions (6) and (7) to compute the variations of profits and surpluses:

$$\begin{aligned}
\Pi^T - \Pi^0 &= -\tfrac{1}{2}\left[\tau_a^v\left(\theta_a - \sigma_a\right) + \tau_b^v\left(\theta_b - \sigma_b\right)\right] \\
V_a^T - V_a^0 &= P_a^0 - P_a^T = -\tau_a^s - \frac{\tau_a^v}{1-\tau_a^v}C_a + \frac{\tau_a^v - \tau_b^v}{1-\tau_a^v}\sigma_b \\
V_b^T - V_b^0 &= P_b^0 - P_b^T = -\tau_b^s - \frac{\tau_b^v}{1-\tau_b^v}C_b + \frac{\tau_b^v - \tau_a^v}{1-\tau_b^v}\sigma_a
\end{aligned}$$

### 3.1 Incidence of specific and ad valorem taxes

We observe from the above three expressions that a *specific tax* on side $k$ ($\tau_k^s > 0$) is entirely borne by agents on side $k$. This result has nothing to do with the two-sidedness of the market. It is due to the Hotelling specification and our assumption that markets remain covered, even after taxes: as demand is inelastic, firms pass any common cost increase entirely to consumers.[8]

[8] In a previous version of this paper, we show that this result holds true when platforms have asymmetric costs. This is because profits depend on the difference of the marginal costs across platforms, and that this difference does not change when platforms are subject to the same specific tax.

We now decompose the impact of an *ad valorem tax*, say on side $a$ ($\tau_a^v > 0$, $\tau_b^v = 0$, $\tau_k^s = 0$). To isolate the contribution of two-sidedness, we analyze the benchmark case where $\sigma_a = \sigma_b = 0$. We find: $\Pi^T - \Pi^0 = -(\tau_a^v \theta_a/2) < 0$, $V_a^T - V_a^0 = -C_a \tau_a^v / (1 - \tau_a^v) \leq 0$, and $V_b^T - V_b^0 = 0$. That is, the platforms and agents on side $a$ share the burden of the ad valorem tax; agents on side $b$ are unaffected. Introducing positive external effects ($\sigma_a, \sigma_b > 0$) challenges these results in several ways. First, the tax burden on side $a$ agents is lowered as their surplus is increased by $\tau_a^v \sigma_b$: $V_a^T - V_a^0 = -C_a \tau_a^v / (1 - \tau_a^v) + \tau_a^v \sigma_b$. Because agents on side $b$ care about the interaction with agents on side $a$ (i.e., $\sigma_b > 0$), the platforms have lower incentives to pass the tax on agents $a$, i.e. to *increase* $P_a$. It may even be the case (e.g., for low $C_a$) that agents $a$ are better off after the tax ($P_a^T < P_a^0$). Second, agents on side $b$ are now negatively affected: $V_b^T - V_b^0 = -\tau_a^v \sigma_a$. Because agents on side $a$ value the interaction with the other group (i.e., $\sigma_a > 0$), the platform is able to increase revenues on side $a$ by lowering its price on side $b$ (so as to attract more side-$b$ users). Yet, exploiting this externality becomes less profitable as the tax reduces the profit that can be made on additional side-$a$ users. This implies that the platforms have lower incentives to reduce $P_b$; hence, $P_b^T > P_b^0$. Finally, what agents $b$ lose is captured by the platforms: $\Pi^T - \Pi^0 = -(\tau_a^v \theta_a/2) + (\tau_a^v \sigma_a/2)$.

### 3.2 Incidence of transaction taxes

Tax authorities may also want to tax the transactions that take place on platforms.[9] Although we do not model explicitly the transactions, we can draw some insights by interpreting $\sigma_k$ as the value that an agent of group $k$ obtains from interacting with any agent of the other group. A transaction tax would thus reduce the strength of the external effects: $\sigma_k$ is replaced with $\left(1 - \tau_k^t\right) \sigma_k$ where $\tau_k^t$ is the percentage transaction tax on side $k$. Abstracting away all other taxes ($\tau_k^s = \tau_k^v = 0$), we find

$$\begin{aligned} \Pi^T - \Pi^0 &= \tfrac{1}{2} \left(\sigma_a \tau_a^t + \sigma_b \tau_b^t\right) \\ V_a^T - V_a^0 &= -\tfrac{1}{2} \sigma_a \tau_a^t - \sigma_b \tau_b^t \\ V_b^T - V_b^0 &= -\tfrac{1}{2} \sigma_b \tau_b^t - \sigma_a \tau_a^t \end{aligned}$$

We observe that transaction taxes benefit platforms: as external effects weaken, agents on either side become less valuable for platforms, which relaxes price competition and increases profits. As for agents, they are hurt by transaction taxes on both sides: the tax on their side directly reduces their utility from interacting on the platform, while the tax on the other side raises their access fee to the platform.

[9] If tax authorities are able to tax transactions, it would be logical to assume that platforms are able to do so as well. It is indeed often the case in reality that platforms set two-part tariffs, adding usage (or per-transaction) fees to the access (or membership) fees that we consider here. However, the present double-Hotelling model is unsuitable to analyze platform competition with two-part tariffs. Indeed, as shown by Armstrong (2006) and Reisinger (2014), the model generates a continuum of equilibria in that case.

**Summary.** Table 1 summarizes our results. For each category of agents and for each type of tax, it indicates whether the incidence is nil (0), positive (+), negative (-) or ambiguous ($\pm$). Recall that we consider so far symmetric taxes levied on symmetric platforms. In this setting, taxes only affect equilibrium prices; participations do not change, which leaves total network effects and transport costs immune to taxes. Hence, taxes have no other welfare effect than transfers between the agents, the platforms, and the tax authority. The magnitude and direction of these transfers differ with the type of tax.

*Specific taxes* are entirely passed to the agents on the side on which they are levied; the agents on the other side and the platforms are left unaffected. *Transaction taxes* hurt agents on both sides and benefit platforms. As for *ad valorem taxes*, the only clear result is that a tax levied on one side hurts the agents on the other side; the taxed agents may benefit from the tax, and so may the platforms. However if platforms benefit from a tax, say, on side $a$ (i.e., if and only if $\theta_a < \sigma_a$), they necessarily suffer from a tax on side $b$ because the second order condition requires $\theta_a + \theta_b > \sigma_a + \sigma_b$, which implies here that $\theta_b > \sigma_b$. Hence, it is always possible to design an ad valorem tax scheme that extracts some profit of the platforms (but this is the sole scheme that does so).

| | $\tau_a^s$ | $\tau_b^s$ | $\tau_a^v$ | $\tau_b^v$ | $\tau_a^t$ | $\tau_b^t$ |
|---|---|---|---|---|---|---|
| $\Pi^T - \Pi^0$ | 0 | 0 | $\pm$ | $\pm$ | + | + |
| $V_a^T - V_a^0$ | - | 0 | $\pm$ | - | - | - |
| $V_b^T - V_b^0$ | 0 | - | - | $\pm$ | - | - |

Table 1: Incidence of taxes for symmetric platforms

# 4 Asymmetric taxes

If platforms are hit by asymmetric taxes, they face asymmetric effective costs. So, we need to solve the price competition game in the presence of cost asymmetries among platforms. Unfortunately, we cannot solve the model with cost asymmetries and ad valorem taxes. We therefore solve the model by setting $\tau_a^v = \tau_b^v = 0$ in the profit functions (3) and (4) and we repeat the analysis of the previous section.

## 4.1 Equilibrium of the asymmetric game

To facilitate the exposition, we define $D \equiv 9\theta_a\theta_b - (2\sigma_a + \sigma_b)(\sigma_a + 2\sigma_b)$, which is positive according to Assumption (5), and $\gamma_k \equiv \tilde{C}_k - \tilde{c}_k$. The equilibrium prices on side $a$ are found as

$$\begin{aligned} P_a^* &= \left[\tilde{C}_a + \theta_a\right] - \sigma_b - \frac{1}{3}\gamma_a - \frac{\sigma_a - \sigma_b}{3D}\left[3\theta_a\gamma_b + (2\sigma_a + \sigma_b)\gamma_a\right] \\ p_a^* &= \left[\tilde{c}_a + \theta_a\right] - \sigma_b + \frac{1}{3}\gamma_a + \frac{\sigma_a - \sigma_b}{3D}\left[3\theta_a\gamma_b + (2\sigma_a + \sigma_b)\gamma_a\right] \end{aligned}$$

The first two components are as above: the Hotelling formula (effective cost plus transport cost) is decreased by the external effect exerted on the other side ($\sigma_b$). Cost asymmetries introduce two additional components. There is first a direct effect: the platform with the higher (resp. lower) cost charges a lower (higher) price. That is, if $\gamma_a$ is positive, meaning that $\tilde{C}_a > \tilde{c}_a$, $P_a^*$ is decreased by $\gamma_a/3$ while $p_a^*$ is increased by the same amount. Second, there is an interplay between cost differences (on both sides) and cross-side external effects. Two comments are in order regarding the latter term. On the one hand, its complexity makes it impossible to carry out any comparative statics exercise with respect to $\sigma_a$ and $\sigma_b$ (and, hence to transaction taxes). On the other hand, this term disappears in the particular case where cross-side external effects are the same across sides ($\sigma_a = \sigma_b$), as we will explain below. Deriving equilibrium prices on side $b$ accordingly, we can now compute the equilibrium participations on the two platforms:

$$\begin{array}{l} N_a^* = \frac{1}{2} - \frac{1}{2D}\left(3\theta_b\gamma_a + (\sigma_a + 2\sigma_b)\,\gamma_b\right),\; n_a^* = 1 - N_a^*, \\ N_b^* = \frac{1}{2} - \frac{1}{2D}\left(3\theta_a\gamma_b + (2\sigma_a + \sigma_b)\,\gamma_a\right),\; n_b^* = 1 - N_b^*. \end{array}$$

To guarantee that the equilibrium participations are strictly positive and lower than unity, we impose the following restrictions on the space of parameters:

$$\left|3\theta_b\gamma_a + (\sigma_a + 2\sigma_b)\,\gamma_b\right| < D \text{ and } \left|3\theta_a\gamma_b + (2\sigma_a + \sigma_b)\,\gamma_a\right| < D. \tag{8}$$

As the equilibrium participations, $N_k^*$, depend on the cost differences, $\gamma_k$, but not on the level of the effective costs, symmetric specific taxes, which increase the effective costs of the two platforms in the same way, do not modify participations. This is what we had in the previous section where taxes affected prices only. The asymmetric tax that we now consider will have an additional impact as it will also affect how agents split across platforms. To disentangle these effects, we perform the following simple exercise: we assume that only platform $U$ is subject to a specific tax only on side $a$; we also assume that platforms have the same costs.

### 4.2 Tax incidence on prices and participations

Before tax, we have $P_a^0 = p_a^0 = C_a + \theta_a - \sigma_b$, $P_b^0 = p_b^0 = C_b + \theta_b - \sigma_a$ and $N_a^0 = N_b^0 = 1/2$. To compute prices and participations after a specific tax $\tau_a$ paid by platform $U$, we just need to set $\gamma_a = \tau_a$, and $\gamma_b = 0$ in the above expressions. Comparing the values before and after tax we find:

$$\begin{array}{ll} P_a^T - P_a^0 = \frac{2}{3}\tau_a - (\sigma_a - \sigma_b)\,\frac{2\sigma_a+\sigma_b}{3D}\tau_a, & P_b^T - P_b^0 = (\sigma_a - \sigma_b)\,\frac{\theta_b}{D}\tau_a, \\ p_a^T - p_a^0 = \frac{1}{3}\tau_a + (\sigma_a - \sigma_b)\,\frac{2\sigma_a+\sigma_b}{3D}\tau_a, & p_b^T - p_b^0 = -\,(\sigma_a - \sigma_b)\,\frac{\theta_b}{D}\tau_a. \end{array} \tag{9}$$

$$N_a^T - N_a^0 = -\frac{3\theta_b}{2D}\tau_a, \text{ and } N_b^T - N_b^0 = -\frac{2\sigma_a + \sigma_b}{2D}\tau_a. \tag{10}$$

To understand how the tax affects equilibrium prices, we take as a benchmark the case where cross-side effects are absent: $\sigma_a = \sigma_b = 0$. In this case, the two sides are independent and any change in the profitability on one side (e.g., the introduction of the tax $\tau_a$) does not affect the profitability on the other side: $P_b^T$ and $p_b^T$ are independent of $\tau_a$ and only side $a$ is affected. In particular, platform $U$ increases its price on side $a$ by two thirds of $\tau_a$ and platform $l$ by one third of $\tau_a$ (as usual in the Hotelling model). The presence of cross-side external effects introduces three additional channels through which a tax affects prices. We call them the 'contamination', 'leverage' and 'ricochet' channels.

The first two channels jointly affect the price that platforms set on side $b$. Take platform $U$. The *contamination channel* pushes $P_b^T$ down. Because agents on side $b$ care about the interaction with agents on side $a$ (i.e., $\sigma_b > 0$), the shock resulting from the increase in $\tau_a$ contaminates side $b$ through the following chain of events: the tax constrains the platform to reduce participation on side $a$, which affects negatively participation and, consequently, revenues on side $b$; the platform then reacts by lowering $P_b^T$ so as to mitigate the propagation. In contrast the *leverage channel* pushes $P_b^T$ up. Because agents on side $a$ value the interaction with the other group (i.e., $\sigma_a > 0$), the platform is able to increase revenues on side $a$ by lowering its price on side $b$ (so as to attract more side-$b$ users). Yet, exploiting this channel becomes less profitable as the tax reduces the margin that can be made on additional side-$a$ users. This implies that platform $U$ has lower incentives to reduce $P_b^T$. The net effect of the previous two channels depends on the balance between $\sigma_a$ and $\sigma_b$: there is more power in the leverage channel for $\sigma_a > \sigma_b$, and in the contamination channel otherwise. So, unless $\sigma_a = \sigma_b$, the tax increase drives the platform to modify its price on side $b$, which, through a *ricochet channel*, induces the platform to adjust $P_a^T$. The shock we consider now is the change in $P_b^T$ instead of the introduction of $\tau_a$; we have the same two transmission channels as before but they now push in the same direction. If (say) $\sigma_a > \sigma_b$, $P_b^T$ goes up and the platform has two reasons to decrease $P_a^T$: the increase in $P_b^T$ not only reduces participation on side $b$ and, thus, on side $a$ (contamination), but also increases the margin on side $b$ and so, the incentive to decrease $P_a^T$ (leverage). Hence, compared to the benchmark, a lower fraction of the tax will be passed on to $P_a^T$. The opposite reasoning can be made when $\sigma_b > \sigma_a$. We can apply the same logic to platform $l$ but in the reverse direction, as the tax makes platform $l$'s cost (relatively) lower. We observe indeed: $p_a^T - p_a^0 = \tau_a - \left(P_a^T - P_a^0\right)$ and $p_b^T - p_b^0 = -\left(P_b^T - P_b^0\right)$.

*In sum*, in the special case where $\sigma_a = \sigma_b$, the tax only affects prices on side $a$: the taxed platform passes 2/3 of the tax on to side-$a$ agents; the other platform reacts by increasing its price by 1/3 of the tax. In comparison, in the case where $\sigma_a > \sigma_b$ (interaction is more valuable for side-$a$ agents), the taxed platform transfers part of the pass-trough from side $a$ to side $b$, while the other platform raises its price further on side $a$ but reduces its price on side $b$. It is important to note that the taxed platform may even choose a form of 'negative pass-through' as its optimum could be to *decrease* its price on side $a$.[10] The opposite situation prevails when $\sigma_b > \sigma_a$. Here (still in comparison with the case $\sigma_a = \sigma_b$), the taxed platform intensifies the pass-through on side $a$ as it chooses to reduce its price on side $b$, while the other platform responds by lessening its price increase on side $a$ and by increasing its price on side $b$. In this case, it is even possible that the other platform ends up decreasing its price on side $a$. Finally we observe that participation unambiguously decreases on both sides for the taxed platform.

[10]An example could be the following: a game platform that is imposed a larger tax on game consoles ends up decreasing the price of its console while increasing the fee it charges to game developers.

### 4.3 Tax incidence on profits

Now that we have a clear mapping of the effects of the tax on the equilibrium prices of the two platforms, let us examine how the price changes translate into profit changes. The change in platform $U$'s profit can be decomposed into three effects (see the appendix for the derivation):

$$\Pi^T - \Pi^0 = -\tau_a N_a^T - \frac{\tau_a}{2D}\left[3\theta_b\left(P_a^T - P_a^0\right) + (2\sigma_a + \sigma_b)\left(P_b^T - P_b^0\right)\right] + \frac{1}{2}\left(p_a^T - p_a^0 + p_b^T - p_b^0\right). \tag{11}$$

The first term is the *direct effect* and is clearly negative: the profit decreases as the platform has to pay the tax $\tau_a$ for all agents it now admits on side $a$ ($N_a^T$). The second term reflects what could be called the *own-price effect*, as it describes how the taxed platform affects its profit by adjusting its own prices. Simple computations show that the own-price effect is always negative. Finally, the third term gives the *strategic effect*: the tax leads the rival platform to modify its prices, which affects in turn the profit of the taxed platform. Using expressions (9), we compute that the strategic effect is equal to

$$SE = \frac{\tau_a}{6} + \frac{\tau_a}{6D}\left(\sigma_a - \sigma_b\right)\left(2\sigma_a + \sigma_b - 3\theta_b\right). \tag{12}$$

Following Fudenberg and Tirole (1984), we expect the strategic effect to be positive because platforms compete in prices. That is, the tax should lead platform $l$ to increase its prices, thereby affecting positively platform $U$'s profit. This is indeed what we obtain in 'one-sided markets', i.e., when cross-side effects are absent, $\sigma_a = \sigma_b = 0$. Otherwise, we see that if $(\sigma_a - \sigma_b)$ $(2\sigma_a + \sigma_b - 3\theta_b) > 0$ (resp. $< 0$), the strategic effect is larger (resp. smaller) than in the case where cross-side effects are nil. This can lead to two striking situations. On the one hand, cross-side effects may make the strategic effect grow so large that it eventually outweighs the direct and own-price effects of the tax; as a result, the platform *benefits* from the introduction of the tax and we talk of a *lucky break*. Indeed, we compute

$$\Pi^T - \Pi^0 = -\frac{1}{2D}\tau_a\left[6\theta_a\theta_b - (2\sigma_a + \sigma_b)(\sigma_a + \sigma_b) + \theta_b(\sigma_a - \sigma_b) - \theta_b\tau_a\right],$$

which can be positive. In contrast, cross-side effects may depress the strategic effect to such an extent that it eventually becomes negative; then, the tax hurts the platform twice, first directly and next through the price reaction of the other platform; there is thus *double jeopardy.*

Regarding the impact of the tax on the other platform's profit, we would expect it to be positive. However, the previous analysis taught us not to trust our hunches. We check indeed that

$$\pi^T - \pi^0 = \frac{1}{2D}\tau_a\left[6\theta_a\theta_b - (2\sigma_a + \sigma_b)(\sigma_a + \sigma_b) + \theta_b(\sigma_a - \sigma_b) + \theta_b\tau_a\right],$$

which can be negative. As for total profits, it turns out that they increase for any tax: $\left(\Pi^T + \pi^T\right) - \left(\Pi^0 + \pi^0\right) = \theta_b\tau_a^2/D > 0$.

To illustrate the tax incidence on profits consider the following values for the parameters: $\theta_a = 25$, $\theta_b = 4$, $\sigma_b = 8$, $\tau_a = 1/2$ and $\sigma_a < 11.87$.[11] For $\sigma_a < 11.61$, the tax hurts platform $U$

[11] By the second order condition (5), the admissible values of $\sigma_a$ are lower than 12. It can be shown that $N_a^T$ and $N_b^T$ are smaller than one whereas $N_a^T > 0$ if $\sigma_a < 11.98$ and $N_b^T > 0$ if $\sigma_a < 11.87$.

and benefits platform $l$, as expected. For $\sigma_a \in (11.61, 11.67)$, both platforms benefit from the tax. Interestingly, for $\sigma_a \in (11.67, 11.87)$, it is the taxed platform that benefits from the tax, while the other platform suffers.

### 4.4 Tax incidence on agents

Total user surplus on side $a$ is defined as:

$$\begin{aligned} V_a \equiv\; & R_a + \underbrace{\sigma_a \left[N_a^* N_b^* + (1 - N_a^*)(1 - N_b^*)\right]}_{TE_a} \\ & - \underbrace{\left[N_a^* P_a^* + (1 - N_a^*) p_a^*\right]}_{TS_a} - \underbrace{\theta_a \left(N_a^{*2} - N_a^* + \tfrac{1}{2}\right)}_{TT_a} \end{aligned}$$

where $TE_a$ denotes the total external benefits, $TS_a$, the total spending, and $TT_a$, the total transport costs. The introduction of the tax affects the three components in contrasting ways. Recalling that $N_a^T < N_a^0$ and $N_b^T < N_b^0$, we necessarily have that $TE_a^T > TE_a^0$ and $TT_a^T > TT_a^0$. As for total external benefits, concentrating more agents of both sides on platform $l$ increases the total number of transactions; for that matter, agents would prefer to be all located on the same platform. The reverse argument applies for total transport costs: they are clearly minimized at $N_a^* = 1/2$. Finally, we cannot ascertain the net effect of the tax on total spending because prices can go any direction (as we have shown above). The total effect of the tax on the surplus is computed as

$$V_a^T - V_a^0 = \left(\frac{9}{2}\frac{\theta_b^2}{D^2}\theta_a \tau_a - 1\right)\frac{\tau_a}{2}$$

which is clearly negative for low values of the tax.

Proceeding accordingly for side $b$, we find

$$V_b^T - V_b^0 = \theta_b \left(\frac{2\sigma_a + \sigma_b}{2D}\tau_a\right)^2$$

which is clearly positive: the introduction of the tax on side $a$ makes agents on side $b$ unambiguously better off.

### 4.5 Welfare impact of the tax

The total amount of tax collected on side $a$ is given by $\tau_a N_a^T$. As $N_a^T$ is linearly decreasing in $\tau_a$, the Dupuit-Laffer curve is bell-shaped, with a peak at $\tau_a = D/(6\theta_b)$. In this asymmetric setting, taxes generate more than monetary transfers from agents and/or platforms to the tax authority; they also affect participations and, thereby, total external benefits and total transport costs. We define social welfare, $W$, as the sum of the surpluses, the profits and the tax revenue. The tax revenue is the sum of the changes in profits and in total spending, which allows us to compute the variation in welfare as:

$$\begin{aligned} W^T - W_0 &= \left(TE_a^T - TE_a^0\right) + \left(TE_b^T - TE_b^0\right) - \left(TT_a^T - TT_a^0\right) - \left(TT_b^T - TT_b^0\right) \\ &= \frac{(2\sigma_a + \sigma_b)(4\sigma_a + 5\sigma_b) - 9\theta_a\theta_b}{4D^2}\theta_b \tau_a^2 \end{aligned}$$

Intuitively, the asymmetric tax is welfare-enhancing if the larger external benefits (which increase with $\sigma_a$ and $\sigma_b$) compensate for the larger total transport costs (which increase with $\theta_a$ and $\theta_b$). Otherwise, the tax is welfare-detrimental; both situations are compatible with the admissible range of parameters.

**Summary.** Table 2 summarizes our results. For each category of agents it indicates whether the incidence of the tax is positive (+) or ambiguous (±). All agents on side $b$ and at least one platform benefit from the specific tax levied on side $a$. Agents on side $a$ may suffer from the tax, and, as a result, the impact of the tax on total welfare is ambiguous.

| $\Pi^T$ | $\pi^T$ | $\Pi^T + \pi^T$ | $V_a^T$ | $V_b^T$ | $W^T$ |
|---|---|---|---|---|---|
| ± | ± | + | ± | + | ± |

Table 2: Incidence of asymmetric specific taxes on side a

# 5 Conclusion

To this date the issue of tax incidence on competing two-sided platforms is largely underexplored. In this paper we have tried to advance our knowledge on this issue within a specific setting. We have highlighted a number of potential counterintuitive effects of taxes. Because platforms set their prices to reflect cross-side external effects, a tax on one side may end up being supported by agents located on the other side. Also, asymmetric taxes may improve welfare by pushing agents on the untaxed platform, which increases the total level of interactions in the market. In our setting, it could also be the case that platforms (even a taxed one) welcome taxes. Arguably, the latter result has a lot to do with the Hotelling framework that we use, and especially the assumption that markets are covered (i.e., that total demand is inelastic). It remains to be seen whether this result could hold in a more general setting where taxation affects total demand. However, the industrial organization literature on two-sided platforms has not produced so far such a general setting which is tractable enough.

Our setting presents other limitations. First, our modeling of transactions among users is very crude. In future research, it would be useful to give a deeper micro-foundation of the users' utilities. Second, in some important platform markets, users on one side multihome and platforms are not able (or allowed) to set negative fees. It would thus be interesting to reconsider our analysis under such features. On the one hand, multihoming modifies the competitive game between platforms: competition is relaxed on the multihoming side and intensified on the singlehoming side. On the other hand, the restriction to non-negative fees may prevent platforms from transferring the burden of a tax from one side to the other.

# 6 Appendix

Recalling that $\Pi = \left(P_a - \tilde{C}_a\right) N_a + \left(P_b - \tilde{C}_b\right) N_b$, we can write

$$\Pi^T - \Pi^0 = -\tau_a N_a^T + \Delta_{P_a} N_a^T + \left(P_a^0 - C_a\right) \Delta_{N_a} + \Delta_{P_b} N_b^T + \left(P_b^0 - C_b\right) \Delta_{N_b},$$

where, using expressions (1) and (2) and the tax incidence on prices,

$$\begin{array}{l}
\Delta_{N_a} \equiv N_a^T - N_a^0 = \frac{\theta_b(\Delta_{p_a} - \Delta_{P_a}) + \sigma_a\left(\Delta_{p_b} - \Delta_{P_b}\right)}{2(\theta_a\theta_b - \sigma_a\sigma_b)}, \\
\Delta_{N_b} \equiv N_b^T - N_b^0 = \frac{\theta_a\left(\Delta_{p_b} - \Delta_{P_b}\right) + \sigma_b(\Delta_{p_a} - \Delta_{P_a})}{2(\theta_a\theta_b - \sigma_a\sigma_b)}, \\
\Delta_{P_a} \equiv P_a^T - P_a^0,\ \Delta_{P_b} \equiv P_b^T - P_b^0, \\
\Delta_{p_a} \equiv p_a^T - p_a^0,\ \Delta_{p_b} \equiv p_b^T - p_b^0.
\end{array}$$

Grouping terms, we have

$$\begin{array}{rcl}
\Pi^T - \Pi^0 & = & -\tau_a N_a^T + \left(N_a^T - \frac{\theta_b\left(P_a^0 - C_a\right)}{2(\theta_a\theta_b - \sigma_a\sigma_b)} - \frac{\sigma_b\left(P_b^0 - C_b\right)}{2(\theta_a\theta_b - \sigma_a\sigma_b)}\right) \Delta_{P_a} \\
& & + \left(N_b^T - \frac{\sigma_a\left(P_a^0 - C_a\right)}{2(\theta_a\theta_b - \sigma_a\sigma_b)} - \frac{\theta_a\left(P_b^0 - C_b\right)}{2(\theta_a\theta_b - \sigma_a\sigma_b)}\right) \Delta_{P_b} \\
& & + \left(\frac{\theta_b\left(P_a^0 - C_a\right)}{2(\theta_a\theta_b - \sigma_a\sigma_b)} + \frac{\sigma_b\left(P_b^0 - C_b\right)}{2(\theta_a\theta_b - \sigma_a\sigma_b)}\right) \Delta_{p_a} \\
& & + \left(\frac{\theta_a\left(P_b^0 - C_b\right)}{2(\theta_a\theta_b - \sigma_a\sigma_b)} + \frac{\sigma_a\left(P_a^0 - C_a\right)}{2(\theta_a\theta_b - \sigma_a\sigma_b)}\right) \Delta_{p_b}.
\end{array}$$

Recalling that $P_a^0 - C_a = \theta_a - \sigma_b$ and $P_b^0 - C_b = \theta_b - \sigma_a$, we find

$$\Pi^T - \Pi^0 = -\tau_a N_a^T + \left(N_a^T - \frac{1}{2}\right) \Delta_{P_a} + \left(N_b^T - \frac{1}{2}\right) \Delta_{P_b} + \frac{1}{2}\Delta_{p_a} + \frac{1}{2}\Delta_{p_b}.$$

which gives the expression in the text after replacing $N_a^T$ and $N_b^T$ by their respective values.

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